





Turns and Turning Flight



Session Time: One, 50-minute session

DESIRED RESULTS

ESSENTIAL UNDERSTANDINGS

The principles of aerodynamics allow an aircraft to fly, yet those same principles limit its ultimate performance and capabilities. (EU2)

Safe and efficient aviation operations require that pilots use math, science, and technology. (EU4)

A deep understanding of how an aircraft operates enables a pilot to fly the aircraft to its maximum capabilities in both normal and abnormal situations. (EU5)

ESSENTIAL QUESTIONS

- 1. What makes an airplane turn?
- 2. What constitutes a good turn?
- 3. Why does an airplane bank in order to turn?

LEARNING GOALS

Students Will Know

- The forces acting on an aircraft in a turn.
- The relationship between turn rate and radius.
- The difference between a slipping and skidding turn.

Students Will Be Able To

- Explain the aerodynamics involved in a turn. (DOK-L2)
- Differentiate between a coordinated turn and an uncoordinated turn. (DOK-L3)
- Calculate the rate of turn in aircraft. (DOK-L2)
- Calculate the radius of turn in aircraft. (DOK-L2)

ASSESSMENT EVIDENCE

Warm-up

Students will watch two videos and answer several questions about the forces occurring on airplanes during turning flight.

Formative Assessment

Students will label a diagram of a turning airplane to explain why it is necessary to increase the angle of attack to avoid losing altitude during a turn, and they will further explain why angle of attack must increase as a turn becomes steeper.

Summative Assessment

Students perform calculations involving a airplane's rate of turn and radius of turn to determine how to safely complete an air race slalom event.

LESSON PREPARATION

MATERIALS/RESOURCES

- Turns and Turning Flight Presentation
- Turns and Turning Flight Student Activity 1
- Turns and Turning Flight Student Activity 2
- Turns and Turning Flight Teacher Notes 1
- Turns and Turning Flight Teacher Notes 2

Rate of Turn and Radius of Turn Equations (per student)

Calculator

Recommended Student Reading

Pilot's Handbook of Aeronautical Knowledge
 Chapter Five, Sections on Aerodynamic Forces in Flight Maneuvers, Rate of Turn, and Radius of Turn
 https://www.faa.gov/regulations_policies/handbooks_manuals/aviation/phak/media/07_phak_ch5.pdf

LESSON SUMMARY

Lesson 1 - Turns and Turning Flight

Lesson 2 - Load Limits in Aircraft Design

In this lesson, students will learn how an airplane turns during flight, with an emphasis on how airplanes make coordinated turns and avoid slipping and skidding. The lesson will begin by watching two videos and answering several questions that will get students thinking about the forces occurring on airplanes during turning flight.

During the next part of the lesson, students will learn what control inputs are required to turn an airplane in flight. The aerodynamics of the turn, including the horizontal and vertical components of lift with respect to an airplane's weight and center of gravity, will be addressed. Next, students will learn about the effects of angle of attack (AOA), angle of bank (AOB), and airspeed on the quality of a turn. In a formative assessment, students will label a diagram of a turning airplane to support their explanation of why it is necessary to increase an airplane's angle of attack to avoid losing altitude during a turn. They will also further explain why the angle of attack must increase as bank angle increases.

Students will also learn about the rate of turns and the radius of turns and the factors that affect both concepts. Students will learn the equations for rate and radius of turn and complete some examples of each.

Finally, students perform calculations involving a airplane's rate of turn and radius of turn to determine how to safely complete an air race slalom event. They will determine how fast they should plan to fly their race airplane around three pylons, how long will it take to fly around each individual pylon, and explain what they could do to traverse the pylons more quickly.

BACKGROUND

To turn an airplane, a pilot uses the control stick or yoke to bank the wings of the airplane in the desired direction. Moving the yoke deflects the ailerons on the wings of the airplane in different directions, which changes the lift generated by each wing. This redistribution of lift causes the airplane to roll in the direction commanded by the pilot. For instance, if the airplane's yoke is turned to the right, the aileron on the right wing deflects upward while the aileron on the left wing deflects downward. The upward deflection of right aileron decreases the camber of the right wing resulting in decreased lift. The downward deflection of the left aileron increases the camber of the left wing resulting in

increased lift. The increased lift on the left wing and decreased lift on the right wing causes the aircraft to roll to the right. The same concepts apply if the pilot moves the yoke to the left, causing the airplane to roll to the left.

The increased lift created by the upward-moving wing also creates more induced drag on that wing. This drag will seem to "pull" the airplane's nose toward the opposite direction of the turn. This undesirable effect is called adverse yaw. Adverse yaw is unavoidable as long as the airplane's wings have different lift gradients. To nullify this effect while turning, pilots use the rudder to point the nose of the airplane as necessary to fly a coordinated path through the turn. A good turn in an airplane uses a combination of ailerons and rudder inputs. This combination results in a coordinated turn. If a turn is uncoordinated, a pilot and passengers may feel pushed sideways toward the outside or inside of the turn. When the push is toward the inside of the turn, the plane is in a slip; when the push is toward the outside, the plane is in a skid. Airplane instruments as well as experience help a pilot determine whether the turn is coordinated or not. Students first learned about coordinated turns in the lesson on primary flight controls (Unit 5, Section B, Lesson 1).

As an airplane banks into a turn, the vertical component of lift that was necessary to counter the weight of the airplane and sustain the altitude is divided into a horizontal component, which pulls the airplane in the direction of turn, and the remainder of the vertical component, which must balance the weight of the airplane to maintain level flight. Typically, this vertical component of lift is insufficient to maintain the airplane's altitude. This is increasingly pronounced as the bank angle is increased. Therefore, a pilot generally has to increase angle of attack (by pulling back on the elevator) to maintain altitude in a turn. Students should recall from our previous discussion on aerodynamics that when angle of attack is increased, everything else being equal, the airplane will begin to slow down. Therefore, more engine power is necessary to maintain airspeed through a level turnthe greater the bank angle, the greater the necessary engine power.

It is clear that an airplane in a turn is subject to multiple inputs of ailerons, rudder, elevator, and power to maintain a level turn. All these inputs are directly proportional to the amount of bank angle used: the greater the bank angle, the greater the input necessary to sustain a level turn.

In practice, turns can be broken down into three basic categories; shallow turns (20° angle of bank or less), medium turns (approximately 20° to 45°), and steep turns (45° or more). The bank angle of an airplane and the speed of the airplane determine the gravitational (G) forces acting on the airplane. (G forces will be covered in greater detail in a later lesson.) Additionally, the speed of the airplane impacts its rate and radius of turn. Generally, for a given angle of bank, the faster the airplane, the lower the rate of turn and the higher the radius of turn. The reverse is also true--the slower the airplane, the higher the rate of turn and the lower the radius of turn.

MISCONCEPTIONS

Students may naturally assume that turning an airplane is fundamentally the same action as turning a car. In fact, turns in a car happen about the car's vertical axis, a motion we call "yaw" in the airplane. We actually turn an airplane about its longitudinal axis, a motion called "roll." Cars have very little difficulty turning about their vertical axis because of ground friction between the tires and the road: the tires will track the desired path, provided the car's speed is not too great, because of their contact with the ground. In contrast, airplanes do not contact the air in this way. So, even through we do steer the airplane on the ground with rudder alone (thanks to ground friction), in the air, we use both the control yoke (controlling ailerons) and the rudder to ensure a smooth, coordinated turn.

DIFFERENTIATION

To support student learning in the **EXTEND** section, walk through examples of calculations and verbalize thoughts while demonstrating how the calculation is to be completed. Provide students additional calculations to perform as practice.

LEARNING PLAN

ENGAGE

Teacher Material: <u>Turns and Turning Flight Presentation</u>

Slides 1-3: Introduce the topic and learning objectives of the lesson.

Slide 4: Conduct the Warm-Up.

Warm-Up

To begin the lesson, students will watch two videos. Have them hypothesize what is happening in each video and then discuss the answers to several questions that will get students to start thinking about the forces occurring on airplanes during turning flight.

The first video features a few race cars and motorcycles speeding around a steeply banked track. Have students observe the action and hypothesize how it is possible for the cars to stay up on the track. You might ask them if the vehicles would be able to sit still on such a steep slope.

 "Cars Driving on a Wall - Centrifugal Force" (Length 1:00) http://video.link/w/f7Ve

The second video features a young boy taking a ride in an aerobatic airplane. Have the students observe what happens when they make a steep turn. Point out how the boy's face is distorted and how he loses consciousness. Have them hypothesize what caused this to happen.

 Kid Feels Effects of 8G's (Length 00:38) http://video.link/w/E7Ve

[DOK 3; hypothesize, DOK 2; explain]

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Questions

After watching both videos, discuss the following questions with the students:

What does it feel like when you go around a sharp corner in a car? Could that have something to do with what is happening in these videos?

Students will likely describe the feeling of getting pushed to the outside of the turn. They might make the connection that those same forces are pulling on the kid and perhaps a connection to the cars being able to stay up on the track.

In both videos, the cars and the airplane are making steeply banked turns. What common factor do you think might be involved?

There are forces acting on both the airplanes that do not happen in level flight or just driving on the ground.

EXPLORE

Teacher Materials: <u>Turns and Turning Flight Presentation</u>, <u>Turns and Turning Flight Teacher Notes 1</u>
Student Material: Turns and Turning Flight Student Activity 1

Slide 5: Have students use a think-pair-share to answer the following review questions related to turning an airplane. If necessary, remind students that turns are basic flight maneuvers used by pilots to change the direction in which an airplane is moving.



Questions

How does a pilot turn an airplane in level flight? What is the primary control surface used?

A turn is accomplished primarily by deflecting the ailerons; pilots do this by moving the yoke to the right or left. Deflecting the ailerons causes the airplane's wings to bank in the direction of the turn.

How does this control surface move when the plane executes a right turn? A left turn?

To make a right turn, a pilot moves the yoke to the right. This action causes the aileron on the right wing to deflect upward and the aileron on the opposite (left) wing to deflect downward. The upward deflection of right aileron decreases the camber resulting in decreased lift on the right wing. The downward deflection of the left aileron increases the camber resulting in increased lift on the left wing. The increased lift on the left wing and the decreased lift on the right wing causes the aircraft to roll to the right. The same concepts apply if the pilot moves the yoke to the left, causing the airplane to roll to the left.

What other control surface is typically involved in making a turn? How?

The rudder typically supplements the ailerons during a turn. The rudder alone is not effective at causing an airplane to turn, since the force generated in flight is not sufficient to significantly change the airplane's heading, or direction of flight. Just like bikes or race cars on banked tracks, the airplane has to "lean" in the direction of its turn: that is, direct some of the lift horizontally to "push" the nose in the direction of the turn, thereby changing the airplane's direction or heading.



Teaching Tips

Students may ask why the rudders aren't used primarily to turn the airplane. The first video from the warm up could help in answering such questions. Point out that the rudder force generated in flight is not sufficient to significantly change the airplane's heading, or direction of flight. Just like bikes or race cars on banked tracks, the airplane has to "lean" in the direction of its turn. This helps to change the lift vector acting upward into the direction of turn and cause the airplane to turn.

Slides 6-7: As a precursor to the forces acting on an airplane in a turn, show students this detailed video that explains how an airplane most efficiently makes a turn in flight.

 How Aircraft Turn (Length 2:16) https://video.link/w/KiRWb



Teaching Tips

It could be helpful to review the forces acting on an airplane in flight as well as Newton's first law of motion, the Law of Inertia:

- The forces acting on an airplane in flight are lift (upward), weight (downward), thrust (forward), and drag (backward). They are taught in Unit 4.
- Newton's first law of motion states that a body at rest remains at rest, and a body in motion remains in motion, unless acted on by an external, unbalanced force. Students also learned about Newton's three laws of motion are taught in Unit 4.

These concepts are related in that level flight occurs when all four forces are balanced. To make a turn, the pilot makes an adjustment to one or more of the primary control surfaces, which unbalances the forces acting on the airplane.

An airplane requires a sideward force to change its lateral flight path. This force is generated by banking the airplane's wings and forcing some of the upward lifting force to act sideways. The vertical component of lift in flight, which originally acted to balance the aircraft's weight in straight-and-level flight, is broken into vertical and horizontal components in a turn. Both forces act at right angles to each other. The vertical component of lift is still necessary to balance the weight of the airplane. This weight is constant but the lift has now been split in two directions; consequently, the vertical component of lift is now insufficient to balance the weight of the airplane and will need to increase to maintain altitude. This is typically done by increasing the angle of attack of the wings by slightly pulling back on the control yoke to deflect the elevator upwards.

The horizontal component of lift, on the other hand, acts in the intended direction of the turn and is responsible for "pulling" the airplane into the turn. This is called centripetal, or "center-seeking," force. The horizontal component of lift also creates an apparent equal and opposing force called the centrifugal force. This force is what creates G-loading on the airplane. Rudder pressure must be applied when entering and exiting the turn to maintain coordinated flight. More on this is covered later in the lesson.

All the forces acting on the airplane in a turn are further pronounced when the angle of bank is increased. Increased bank angle results in increased loads on the airplane; therefore, increased lift is necessary to keep the airplane level. As the airplane produces more lift to stay level, more induced drag is also generated, which slows the airplane down. Thus, more power will also be required to maintain desired airspeed.

Slide 8: Complete the Formative Assessment.

Formative Assessment

In this formative assessment, students will label a diagram of a turning airplane to support their explanation of why it is necessary to increase an airplane's angle of attack to avoid losing altitude during a turn. They will also further explain why the angle of attack must increase as bank angle increases.

Distribute the Turns and Turning Flight Student Activity 1 to students. Sample answers are provided in the Turns and Turning Flight Teacher Notes 1.

[DOK 2; explain]

EXPLAIN

Teacher Material: Turns and Turning Flight Presentation

Slide 9: Airplanes can be turned using varying angles of bank (AOB), depending on the pilot's requirement and the phase of flight. Turns are generally classified into shallow, medium, and steep angles of bank. Shallow turns are made using 20° AOB or less. Medium turns have between 20°-44° AOB, while steep turns are made with 45° or greater AOB. Point out that as the angle of bank increases, the horizontal component of lift increases and the turn happens faster.

Slide 10: This and the following three slides review adverse yaw and coordinated and uncoordinated turns, which students learn about first in Unit 5, Section B, Lesson 1.

Adverse yaw is generated any time an airplane is banked. It is the natural and undesirable tendency for an aircraft to yaw in the opposite direction of a roll. Adverse yaw is caused by the lift differential, and consequently drag differential, between the wings. Whenever the pilot moves the control yoke left or right in an airplane, the aileron corresponding to the direction of movement deflects upward while the opposite aileron is deflected downwards. For instance, moving the control yoke left moves the left aileron up and the right aileron down.

The upward deflection of the left aileron reduces the lift produced by the left wing. The opposite happens with the right aileron; the downward deflection increases camber and increases lift on the right wing. The airplane rolls into a left turn. However, the increased lift on the right wing also increases the drag on the wing, slowing down that wing in relation to the opposite (left) wing. This causes the nose of the airplane to yaw toward the wing with increased lift and drag (right).

Adverse yaw is countered by using the rudder (in almost all cases, stepping on the rudder *into* the turn). By adding rudder input, the pilot is creating a side force on the vertical tail that opposes adverse yaw. This creates a yawing moment that helps turn the airplane in the correct direction.

Slide 11: In a coordinated turn, the adverse yaw generated is correctly countered and the airplane does not slip or skid during the turn. The ailerons generate the sideward force to turn the airplane but also create a lateral acceleration in the axis of the turn. The rudder helps counter this lateral acceleration. A person sitting in an airplane during a coordinated turn feels no sideways force.



Questions

Ask students what control surfaces a pilot used to perform a coordinated turn?

To counteract adverse yaw, rudder pressure must be applied simultaneously with aileron in the desired direction of turn. For example, if a pilot is performing a left turn, they would turn the yoke to the left and simultaneously apply left rudder to counter the adverse yawing moment.

If time allows, show students an AOPA video of footage from inside a cockpit to illustrate how to make coordinated turns.

 "Coordinated Turns" (Length: 3:48) http://video.link/w/Z5Me

Slide 12: As students learned previously, in an uncoordinated turn, the lateral acceleration created by the banked ailerons causes the airplane's nose to point either too far outside or inside the turn. To a person sitting in the airplane, there is a feeling of sliding inside or outside the turn, depending on the nature of the uncoordinated turn. An uncoordinated turn can either be a slip or a skid.

In a slipping turn, the tail of the airplane is inside the turn and the nose of the airplane falls to the outside of the turn. This is because the horizontal component of lift generated is greater than the centrifugal force. This force pulls the

airplane further into the turn at a rate slower than the angle of bank used to turn the airplane. A person sitting in an airplane during a slip feels pushed to the inside of the turn. The slip can be countered by decreasing the AOB using the necessary rudder input during the turn.

In a skidding turn, the tail of the airplane is outside the turn and the nose of the airplane falls to the inside of the turn. This is a result of increased centrifugal force relative to the horizontal component of lift. The centrifugal force prevents the airplane from turning at a rate commensurate to the AOB used. A person sitting in an airplane during a skid feels pushed to the outside of the turn. A skid is countered by increasing the AOB and using the necessary rudder input.

EXTEND

Teacher Material: Turns and Turning Flight Presentation

Slide 13: The rate of turn of an airplane refers to how fast an airplane will make a given turn. Rate of turn is determined by the airplane's AOB and speed, and it is measured in number of degrees per second. An airplane's rate of turn equals the number of degrees of heading change that the airplane makes during a turn, and it is inversely proportional to the airplane's airspeed: the higher the airspeed, the lower the rate of turn for a given angle of bank. The horizontal component of lift also increases with an increase in bank angle and consequently increases the rate of turn.

Thus, an airplane can be turned faster in several ways: using a steeper bank angle, using a slower airspeed, or using both (within practical limits of the airplane).

Slide 14: Ensure students have calculators to determine the rate and radius of turns in the following problems.

The rate of turn of an airplane can be calculated using the following formula:

1,091 tangent of the bank angle airspeed in knots

Walk students through the example calculations on the slide.

What is the rate of turn of an airplane at 30° angle of bank and 100 knots?

$$1,091 \text{ tangent } 30^{\circ} = 6.30 \text{ degrees per second}$$

What is the rate of turn of an airplane at 45° angle of bank and 100 knots?

$$1,091$$
 tangent 45° = 10.91 degrees per second

After students complete the first two calculations, ask them how bank angle is related to the rate of a turn for a constant airspeed. The answer is that higher bank angles result in higher rates of turn, assuming airspeed is constant.

Slide 15: Then walk students through the example calculations on the slide which assumes a higher constant airspeed for the same angles of bank provided in the previous examples.

What is the rate of turn of an airplane at 30° angle of bank and 120 knots?

$$1,091 \text{ tangent } 30^{\circ} = 5.25 \text{ degrees per second}$$

What is the rate of turn of an airplane at 45° angle of bank and 120 knots?

$$1,091 \text{ tangent } 45^{\circ} = 9.10 \text{ degrees per second}$$

After students complete these next two calculations, ask them what happens to the rate of turn when an airplane is flown at a higher airspeed for a constant angle of bank. The second two examples demonstrate that greater airspeed results in lower rates of turn, assuming AOB is constant.



Teaching Tips

If time allows, create additional sample problems by substituting different angles of bank and airspeeds into the examples provided.

Slide 16: The radius of turn of an airplane can be understood as the amount of room an airplane needs to make a turn. Imagine a small Piper Cub airplane making a 180° turn compared to a Boeing 737 making the same turn. The radius of turn is directly affected by the airplane's airspeed and its bank angle. The faster an airplane is moving, the greater its turn radius will be; conversely, the slower an airplane, the smaller its turn radius. Note that the same airplane will have a wider radius of turn at higher airspeeds.

Slide 17: The radius of turn of an airplane can be calculated using the following formula. Remind students that airspeed, v, is measured in knots:

$$v^2$$
 11.26 tangent of bank angle

Walk students through the example calculations on these slides.

What is the radius of turn of an airplane at 30° angle of bank and 100 knots?

$$\frac{100^2}{11.26 \text{ tangent } 30^\circ}$$
 = 1,538 feet

What is the radius of turn of an airplane at 45° angle of bank and 100 knots?

$$\frac{100^2}{11.26 \text{ tangent } 45^\circ}$$
 = 888 feet

The first two examples demonstrate that a steeper bank angle results in a smaller radius of turn, assuming airspeed is constant.

After students complete these next two calculations, ask them how bank angle affects the radius of a turn. Students should say that a steeper bank angle results in a smaller radius of turn.

Slide 18: Then walk students through the example calculations on the slide which assumes different airspeeds for a constant angle of bank.

What is the radius of turn of an airplane at 30° angle of bank and 200 knots?

$$200^2$$
 = 6,152 feet 11.26 tangent 30°

What is the radius of turn of an airplane at 30° angle of bank and 100 knots?

$$100^2$$
 = 1,538 feet 11.26 tangent 30°

Ask students how airspeed affects radius of turn for a constant angle of bank. They should realize that an airplane moving at a greater airspeed requires a larger radius of turn, assuming AOB is constant. Point out that doubling airspeed, from 100 to 200 knots, causes the radius of turn to increase by nearly 6 times.

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Teaching Tips

If time allows, create additional sample problems by substituting different angles of bank and airspeeds into the examples provided.

Slides 19–20: Summarize for students that rate of turn (ROT) is directly related to bank angle and inversely related to airspeed; in contrast, radius of turn (R) is inversely related to bank angle and directly related to airspeed.

EVALUATE

Teacher Materials: <u>Turns and Turning Flight Presentation</u>, <u>Turns and Turning Flight Teacher Notes 2</u> Student Material: <u>Turns and Turning Flight Student Activity 2</u>

Slides 21-22: Quiz the students on a question related to turning flight for the Private Pilot Knowledge Test.

Slide 23: Complete the Summative Assessment.

Summative Assessment

In this summative assessment, students perform calculations involving a airplane's rate of turn and radius of turn to determine how to safely complete an air race slalom event. They will determine how fast they should plan to fly their race airplane around three pylons, how long will it take to fly around each individual pylon, and explain what they could do to traverse the pylons more quickly.

Provide students with Turns and Turning Flight Student Activity 2. Answers are provided in the Turns and Turning Flight Teacher Notes 2.

[DOK 1; calculate, DOK 2; explain, DOK 1; recall]

Summative Assessment Scoring Rubric

- Follows assignment instructions
 - Students write the equations correctly
 - Students show all math calculations and calculate the equations correctly
 - Students provide thorough explanation of how to fly through the pylons more quickly
- Contributions show understanding of course of the concepts covered in the lesson
- Contributions show in-depth thinking including analysis or synthesis of lesson objectives

Points Performance Levels

- 9-10 The student correctly shows the radius of turn and rate of turn equations, and finds the correct velocity (knots) and rate (per pylon). All calculations are shown. The student gives a correct and thorough explanation of how to traverse the pylons in a shorter period of time.
- 7-8 The student correctly shows the radius of turn and rate of turn equations. Computational errors prevent the student from getting one or both answers correct. The student gives an good explanation of how to traverse the pylons in a shorter period of time.
- 5-6 The student shows the radius of turn and rate of turn with minor errors. Computational errors prevent the student from getting one or both answers correct. The student's explanation of how to traverse the pylons in a shorter period of time is incorrect.
- O-4 The student shows the radius of turn and rate of turn equations with one or both incorrect. Computations are incorrect or incomplete. The student's explanation of how to traverse the pylons in a shorter period of time is incomplete or incorrect.

STANDARDS ALIGNMENT

NGSS STANDARDS

• **HS-PS2-2** - Use mathematical representations to support the claim that the total momentum of a system of objects is conserved when there is no net force on the system.

Science and Engineering Practices

- o Using Mathematics and Computational Thinking
- o Analyzing and Interpreting Data

Disciplinary Core Ideas

- o PS2.A Forces and Motion
- o PS2.B Types of Interactions

Crosscutting Concepts

o Cause and Effect

COMMON CORE STATE STANDARDS

- RST.9-10.1 Cite specific textual evidence to support analysis of science and technical texts, attending to the precise details of explanations and descriptions.
- RST.9-10.2 Determine the central ideas or conclusions of a text; trace the text's explanation or depiction of a complex process, phenomenon, or concept; provide an accurate summary of the text.
- RST.9-10.4 Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 9-10 texts and topics.
- WHST.9-10.9 Draw evidence from informational texts to support analysis, reflection, and research.

REFERENCES

http://avstop.com/ac/flighttrainghandbook/turns.html

http://www.boldmethod.com/cfi-tools/forces-in-a-turn/

http://www.boldmethod.com/learn-to-fly/aerodynamics/slip-skid-stall/

https://www.faa.gov/regulations_policies/handbooks_manuals/aviation/phak/media/07_phak_ch5.pdf